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# NONLINEARITIES AND POMERON NONFACTORIZABILITY IN CONVENTIONAL DIFFRACTION

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## **Abstract**

Alternatives for describing the nonlinear behavior of the first diffraction cone in differential  $pp$  and  $\bar{p}p$  elastic cross-section are investigated. High quality fits to the data are presented. We show that the presence in the Pomeron amplitude of two terms with different  $t$  dependences is strongly suggested by the data, hinting at a non-factorizable Pomeron even in the field of purely hadronic reactions. The available data, however, do not allow to choose among a nonlinearity in the residues or in the Pomeron trajectory or in both. In all cases, we find an effective slope of the trajectory larger than the one currently used. A nonlinear trajectory with the fitted parameters is used for predicting the mass and the width of the  $2^{++}$  glueball. An excellent agreement is found with the X(1900) candidate from the WA91 experiment.

## 1. Introduction

The major efforts about the Pomeron problem seem to be now concentrated in the Deep Inelastic Scattering (DIS) physics (see for example the new experiments at HERA and their theoretical interpretations in [1]). However, at the same time, the rôle of the Pomeron in the "old" hadronic high energy physics is still theoretically investigated. The subject is now revitalized by projected experiments, such as the PP2PP [2] and TOTEM [3] projects at RHIC and LHC. The reason of this two-fold effort and interest comes mainly from the realization that low -  $x$  physics is probably strongly related to high energy low  $-p_t$  physics, through a unique mediator: the Pomeron, which can be qualified as "soft" or "hard", depending on which aspect of this object one wants to focus.

This paper is concerned with the manifestations of the Pomeron in high energy  $pp$  and  $\bar{p}p$  elastic scattering (see the reviews [4a] and references therein). Many efforts (for example [4b]) have been devoted to the construction of the related amplitude, able to describe the data in a wide range of centre-of-mass energies ( $\sqrt{s}$ ) and 4-momentum transfer (squared) ( $t$ ) in the process. However, it is well-known that, in the intermediate and large  $t$ -region, the Pomeron does not contribute alone to the process : in that kinematical region where the angular distributions exhibit a "dip-bump" structure followed by a "second cone", an interference of several terms (Pomeron, Odderon, Reggeons....) and multiple exchanges between them enter into the game (mainly multiple Pomeron and Odderon exchanges). To clarify, the contribution of the bare Pomeron in the amplitude can be constructed as an input -or Born approximation- which dominates at  $t = 0$  and at small  $|t|$ , corresponding to the "first cone". It is not excluded that the determination of the Pomeron contribution from an overall  $|t|$  consideration is strongly biased by the intermediate and large  $|t|$  data, where multiple exchanges interfere and the relative contribution of the bare Pomeron is hindered. Thus, in our opinion, a clear signal of the Pomeron can be seen at high energy and small  $|t|$  and it is therefore important to analyse first of all this kinematical region.

The data on the first cone were analyzed long ago in a wide energy region. The subsequent appearance of the first dip and second cone slightly shifted the focus of the studies. However, since this pioneering times, many precise measurements of the angular

distribution have been performed at small  $|t|$ , and it is necessary to come back to the original first goal of analysing the elastic scattering data to see what more we can learn from high quality fits to high precision experiments.

Our strategy for the construction of a complete model aiming to reproduce the available experimental data step by step, in a wide domain of  $\sqrt{s}$  and  $t$  will be the following :

1. First we select, as input Pomeron and preasymptotic  $f$ - and  $\omega$ -Reggeons amplitudes, a simple parametrization fitting the  $t = 0$  data and we fix the relevant parameters.
2. Next, we extend the analytical expression for scattering amplitude to take into account the small  $|t|$  -region. From a fit to the corresponding data, we determine the new parameters controlling the behavior of the amplitude in that region.
3. Finally, we incorporate in the total amplitude  $A(s, t)$  terms which are important at  $|t| > 1 \text{ GeV}^2$  and adjust the corresponding new parameters to the data in the intermediate and high  $|t|$ -range, keeping unchanged the part of the amplitude and the parameters determined previously.

This procedure can be repeated if necessary. We mean that, to perform a fine tuning of all the parameters, giving the best fit to the data, it is possible, on a given step, to allow a weak variation of the parameters obtained from the previous step. In our opinion, such a minimization procedure improve our understanding of the physical meaning of each term introduced phenomenologically in the amplitude.

The first step has been performed in [5], where we have considered four models of the Pomeron at  $t = 0$  and obtained the corresponding parameters. In the present paper, we continue the above proposed program , performing the second step, and investigating  $pp$  and  $\bar{p}p$  elastic scattering at small  $|t|$ , in the region of the first diffraction cone. We come to the remarkable conclusion that a non-factorizable Pomeron is more suitable for describing this region than a factorizable one. This, while not ruled out on physical grounds came somewhat of a surprise since most attempts of fitting elastic data have assumed factorizability of the Pomeron. The privilege of our present investigation however resides in the high quality fits we can produce.

In the next section, we give theoretical and phenomenological arguments in favor of a nonlinear Pomeron trajectory. In the section 3, we define the scattering amplitudes and the set of chosen experimental data. In the section 4, we discuss the obtained results of our fit to the data, together with the properties of the Pomeron trajectories under consideration,

yielding predictions for the mass and the width of the first lightest glueball candidate associated with the Pomeron. A summary of our conclusions is presented in a last section.

## 2. Nonlinear Pomeron trajectory

It is well known that the Reggeon trajectories  $\alpha(t)$  have thresholds related to the physical thresholds of the amplitude in the  $t$ -channel [6]. Above a threshold, the trajectory has an imaginary part related to the width  $\Gamma_R$  of a resonance peak at the point  $t_R$  where the real part of trajectory equals the spin  $S$  of the resonance

$$\Re \alpha(t_R) = S. \quad (1)$$

Then, in accordance with the Breit-Wigner formalism,

$$\Gamma_R = \frac{\Im \alpha(t_R)}{\Re \alpha'(t_R) M_R}, \quad M_R = \sqrt{t_R}, \quad (2)$$

where  $\alpha'(t_R)$  is here the derivative taken at the resonance and  $M_R$  is the "mass" associated to the resonance. It is necessary to emphasize that such a trajectory cannot be a linear function of  $t$ . Moreover there are several arguments [7] to think that trajectory  $\alpha(t)$  cannot linearly increase with  $t$ , *i.e.*

$$|\alpha(t)/t| \rightarrow 0 \quad \text{when} \quad |t| \rightarrow \infty. \quad (3)$$

This is certainly valid for the trajectories with resonances. As for the Pomeron, up to now it is not clear whether resonances exist or not. From QCD arguments as well as in analogy with Reggeons, one expects that observable particles (glueballs) should be found on the Pomeron trajectory for integer spins larger than one. Candidates for this role of glueballs are seen in the experiments [8], but the question is still open (some claiming experimental evidence, others claiming the absence of clear experimental signal). Nevertheless, if such resonances exist, the Pomeron trajectory should be nonlinear. It has thresholds and the corresponding minimal  $t$ -value is related to the lightest mass of hadronic state with vacuum quantum numbers. This is a two pions state. So the Pomeron trajectory should have a threshold at  $t_P = 4m_\pi^2$ .

Additional arguments in favor a nonlinear trajectory (for  $t$  not close to zero) can be found also in [9]. In that paper, it has been shown that the unitarity inequality

$$\Im H(s, b) > 0 \quad (4)$$

for an elastic scattering amplitude  $H(s, b)$  in the impact-parameter ( $b$ ) representation as well as the correct asymptotic behavior of this amplitude

$$H(s, b) \sim \exp(-b/b_0), \quad b_0 = \text{constant}, \quad \text{when } b \rightarrow \infty \quad (5)$$

cannot be satisfied, even after an eikonalization has been performed, if the input Pomeron has a linear trajectory.

Finally, we must note that a nonlinearity of the Pomeron trajectory  $\alpha_P(t)$  is to be reflected into the curvature of differential cross-sections ( $\frac{d\sigma}{dt}$ ) at very high energy ( $\sqrt{s}$ ) and small squared transfer ( $|t|$ ). For the linear case ( $\alpha_P(t) = \alpha_P(0) + \alpha'_P t$ ), the local slope  $B(s, t)$  of  $\frac{d\sigma}{dt}$  is  $t$ -independent and of course the local curvature  $C(s, t)$ , related to the second derivative, is zero.

$$B(s, t) = \frac{\partial}{\partial t} \left( \ln \frac{d\sigma(s, t)}{dt} \right) \simeq 2 \alpha'_P \ln \frac{s}{s_0}, \quad C(s, t) = \frac{1}{2} \left( \frac{\partial}{\partial t} B(s, t) \right) = 0, \quad (6)$$

while for nonlinear trajectories, one obtains for the slope and the curvature at very high energy

$$B(s, t) \simeq 2 \frac{d\alpha_P(t)}{dt} \ln \frac{s}{s_0}, \quad C(s, t) \simeq \frac{d^2 \alpha_P(t)}{dt^2} \ln \frac{s}{s_0}. \quad (7)$$

If the rising of a nonlinear  $\alpha_P(t)$  with  $|t|$  is slower than  $t$ , then the second derivative is positive *i.e* at a sufficiently large  $s$ , the curvature will be positive. However, at the available energies, below  $\sqrt{s} = 1.8$  TeV,  $C$  is found positive and at  $\sqrt{s} = 1.8$  TeV,  $C$  is roughly zero [10]. Thus in accordance with [10] the Tevatron energy is close to the transition between the preasymptotic ( $C > 0$ ) and asymptotic ( $C < 0$ ) behaviors of  $\frac{d\sigma(s, t)}{dt}$ . Our model predicts another behavior of  $C$ ; it stills positive and rising with energy. Future experiments, we hope, can help to select the realistic model.

On the other hand, given that the elastic scattering amplitude does not have a singularity at  $t = 0$ , the Pomeron trajectory also must have a regular behavior at  $t = 0$ , if the Pomeron is an isolated singularity in the  $j$ -plane. This is not the case for models in which the Pomeron is a pair of moving cuts colliding at  $t = 0$  (for example, in an eikonalized

model with as input a simple pole having an intercept larger than 1). We note that if the Pomeron trajectory is assumed to behave linearly ( $\alpha_P(t) \approx \alpha_P(0) + \alpha'_P t$ ) when  $t \approx 0$ , the Pomeron pole cannot be harder than a double one. In fact, let the elastic partial amplitude have the pole with arbitrary hardness  $\mu + 1$

$$a(j, t) \sim \frac{1}{(j - 1 - \alpha'_P t)^{\mu+1}} . \quad (8)$$

One can easily obtain the following asymptotic behaviors of the total and elastic cross-sections :

$$\sigma_{tot}(s) \sim \ln^\mu(s/s_0), \quad \sigma_{el}(s) \sim \ln^{2\mu-1}(s/s_0), \quad s_0 = 1 \text{ GeV}^2 . \quad (9)$$

Because  $\sigma_{el} < \sigma_{tot}$ , we must have  $\mu \leq 1$ . In other words, if the Pomeron trajectory is linear for  $t$  close to zero, then the Pomeron cannot have a higher singularity than a double  $j$ -pole.

Thus, we prefer to limit our present analysis and description of the available experimental data to a double Pomeron pole ( $\mu = 1$ ) as well as to a simple Pomeron pole ( $\mu = 0$ ) and use a nonlinear Pomeron trajectory. In addition, we consider also the standard linear case in order to compare different possibilities and see the effects of nonlinearity of the trajectory and of the residue functions (see below). The choice of a common set of experimental data is crucial for discussing the meaning of this comparison.

### 3. Definition of the amplitude and choice of experimental data

#### *Models for the scattering amplitude*

As usual in the Regge approach, we write the following form for the  $\bar{p}p$  and  $pp$  elastic scattering amplitudes

$$A_{pp}^{\bar{p}p}(s, t) = P(s, t) + f(s, t) \pm \omega(s, t) + C_{pp}^{\bar{p}p}(s, t) , \quad (10)$$

where  $P(s, t)$ ,  $f(s, t)$ ,  $\omega(s, t)$  are respectively the Pomeron, the  $f$ - and the  $\omega$ -Reggeon contributions;  $C_{pp}^{\bar{p}p}(s, t)$  is the standard Coulomb amplitude, which has been calculated according to the procedure by West and Yennie [11]. We use the following normalization for the dimensionless amplitude

$$\sigma_{tot} = \frac{4\pi}{s} \Im A(s, t), \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2 . \quad (11)$$

The  $f$ -Reggeon has a standard parametrization

$$f(s, t) = g_f \tilde{s}^{\alpha_f(t)} e^{b_f t}, \text{ with } \alpha_f(t) = \alpha_f(0) + \alpha'_f t. \quad (12)$$

Here and in what follows  $\tilde{s} = -is/s_0$  and  $s_0 = 1 \text{ GeV}^2$ . In the  $\omega$ -Reggeon contribution

$$\omega(s, t) = i g_\omega [1 + t(z_1 + z_2 \ln \tilde{s})] \tilde{s}^{\alpha_\omega(t)} e^{b_\omega t}, \text{ with } \alpha_\omega(t) = \alpha_\omega(0) + \alpha'_\omega t. \quad (13)$$

we insert an additional factor  $1 + t(z_1 + z_2 \ln \tilde{s})$  which phenomenologically describes the cross-over effect, *i.e.* the crossing of the  $\bar{p}p$  and  $pp$  differential cross-sections at some transfer  $t = t_\omega < 0$ . This is illustrated in Fig.1 for  $\sqrt{s} = 13 \text{ GeV}^2$ . Comparing the different possibilities to describe this phenomenon, we observe that the best description of the data is obtained when  $t_\omega$  is moving down to zero when the energy increases. This is a fine effect, however ignoring it leads to a higher value of the  $\chi^2$  (see the numerical results in the next section).

Choosing the Pomeron contribution is a more delicate and complicated subject. One of our aims in the present study is to investigate the effects of nonlinearity of the Pomeron trajectory. However, the nonlinear  $t$ - behavior of  $\ln \frac{d\sigma}{dt}$  can eventually be described by residue functions  $F^2(t)$  which can differ from usual ones  $\exp(b_0 t)$  with  $b_0$  constant. For that reason, we study a more general parametrization  $F^2(t) = \exp \phi(t)$  (see below).

We consider two types of Pomeron singularity, first a double  $j$ -pole with trajectory of unit intercept; we call it "model A" :

$$\textbf{model A} \quad \alpha_P(0) = 1, \quad P(s, t) = [g_0 F_0^2(t) + g_1 F_1^2(t) \ln \tilde{s}] \tilde{s}^{\alpha_P(t)}, \quad (14)$$

where, in order to limit the free parameters, we choose the residue functions

$$F_1^2(t) = \exp(\beta t) F_0^2(t), \quad (15)$$

$$F_0^2(t) = \exp(\phi(t)), \quad \phi(0) = 0. \quad (16)$$

Our second choice will be a simple  $j$ -pole with an intercept larger than 1; we call this model of supercritical Pomeron "model B" :

$$\textbf{model B} \quad \alpha_P(0) = 1 + \Delta > 1, \quad P(s, t) = [g_0 F_0^2(t) \tilde{s}^{-\Delta} + g_1 F_1^2(t)] \tilde{s}^{\alpha_P(t)}, \quad (17)$$

with the same simplifivative choice for the residue functions as in the preceeding case. Here, we would like to note that both expressions (14) and (17) for the Pomeron contribution



cannot satisfy the factorization condition (if  $\beta \neq 0$ ). This is due to the preasymptotic terms  $g_0 F_0^2 \tilde{s}^\alpha$ . For "model A" for example, the first and second terms correspond to the case of a single and double  $j$ -pole respectively. Of course, the double pole contribution taken alone (as the leading term at high energy) would satisfy the factorization condition. Thus, the condition  $\beta \neq 0$  can be considered as an indication of the nonfactorizability of the Pomeron at present energies (and this will, indeed, be one of the conclusions following our analysis, see below). For both models of the Pomeron, we consider and compare a few possibilities (see also [12]) :

(i)

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t + \gamma_P \left(1 - \sqrt{1 - t/t_P}\right), \quad (18)$$

$$\phi(t) = b_0 t + \gamma_0 \left(1 - \sqrt{1 - t/\tau_0}\right) \quad (19)$$

(ii)

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t + \gamma_P \left(1 - (1 - t_P/t) \ln(1 - t/t_P)\right). \quad (20)$$

$$\phi(t) = b_0 t + \gamma_0 \left(1 - (1 - \tau_0/t) \ln(1 - t/\tau_0)\right), \quad (21)$$

(iii)

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t - \gamma_P \ln \frac{1 + \sqrt{1 - t/t_P}}{2}, \quad (22)$$

$$\phi(t) = b_0 t - \gamma_0 \ln \frac{1 + \sqrt{1 - t/\tau_0}}{2}, \quad (23)$$

where in accordance with arguments given in the previous section  $t_P = 4m_\pi^2$ .

The following remarks are in order :

- For simplicity, the nonlinear corrections have the same form in the trajectories and in the function  $\phi(t)$  defining the common factor in the residues.

- In all cases, we write the linear term only as an effective contribution of  $N$  other nonlinear terms, related with the next thresholds, because we consider a very limited region of small  $|t|$ . More generally, for example, in case (i), we could have written instead of (18)

$$\alpha_P(t) = \alpha_P(0) + \sum_{k=1}^N \gamma_{P,k} (1 - \sqrt{1 - t/t_{P,k}}), \quad 4m_\pi^2 = t_{P,1} < t_{P,2} < \dots, \quad (24)$$

so that the effective slope for the linear part of trajectory (8) would be

$$\widetilde{\alpha'_P} = \sum_{k=2}^N \frac{\gamma_{P,k}}{2t_{P,k}}. \quad (25)$$

- The slope of nonlinear Pomeron trajectories depends on  $t$ ; in particular at the origin

$$\widetilde{\alpha'_P} = \left[ \frac{d\alpha_P(t)}{dt} \right]_{t=0} = \alpha'_P + \frac{\gamma_P}{2\nu t_P} \quad (26)$$

where  $\nu = 1$  for (18) or (20) and  $\nu = 2$  for (22).

- The trajectory (ii) does not have an unwanted singular behavior at  $t = t_P$  like the simplest logarithmic trajectory  $\alpha(t) = \alpha(0) - \gamma \ln(1 - t/t_P)$  (the pole of the factor  $1 - t_P/t$  being cancelled by the zero of the logarithm).

- For the purpose of comparison we have also considered the simple case of linear functions

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t, \quad (27)$$

$$\phi(t) = b_0 t. \quad (28)$$

### *Experimental data*

In order to determine the parameters which control the  $s$ -dependence of  $A(s, 0)$ , we use practically the same set of data on the total cross-sections  $\sigma_{tot}(s)$  and on the ratios  $\rho(s) = \Re A(s, 0)/\Im A(s, 0)$  as in [5] for  $\sqrt{s} \geq 5$  GeV. The recent value of  $\sigma_{tot}^{\bar{p}p}$  at the Tevatron energy [13] is added; however, the ancient result  $\rho_{\bar{p}p} = 0.24 \pm 0.04$  at 546 GeV has been eliminated. A total of 208 data has been included for  $t = 0$ .

For the differential cross-sections ( $d\sigma/dt$ ) we have selected the data for energies  $\sqrt{s} > 9$  GeV. However, the angular distributions at the Tevatron have not been taken into account in the fit. The squared 4-momentum has been limited by  $|t| \leq 0.5$  GeV<sup>2</sup>, because at larger  $|t|$  the influence of the dip-region becomes visible (in particular, it can be seen quite clearly at the Collider energy  $\sqrt{s} = 546$  GeV). To be more precise, the  $t$ -limit of the first diffraction cone changes weakly with energy. We think that the chosen region  $|t| \leq 0.5$  GeV<sup>2</sup> is certainly the region of a first cone for the energies investigated here, while at smaller energies it can be extended. Several experiments, in which  $d\sigma/dt$  is measured at small  $|t|$ , have been reported. In the present analysis, we keep only those for which the data cover the largest range of energies and (or) those for which data on both  $pp$  and  $\bar{p}p$  are available. We include also the data for  $d\sigma/dt$  in the Coulomb-nuclear interference region, when they exist. A grand total of 1288 data have been used in the overall fit.

## 4. Results and discussion

### Data at $t = 0$ .

The parameters from the fit to the data at  $t = 0$  are given in Table 1 for both versions of the Pomeron model. We notice that the double pole Pomeron with  $\alpha_P(0) = 1$  (model "A") gives a marginally better  $\chi^2$  than the supercritical Pomeron with  $\alpha_P(0) > 1$  (model "B").

The theoretical total cross-sections  $\sigma_{tot}(s)$  and ratios  $\rho(s)$  are compared to the data for  $pp$  and  $\bar{p}p$  elastic scattering respectively in Fig.2 and Fig.3 for the double pole (model "A") only. The results for both versions would be undistinguishable on these figures except at the Tevatron energy. At such an energy, and consequently at higher energies, the supercritical model predicts a total cross-section greater than the double pole model does (for example at the 14 TeV of the LHC : 110 mb instead of 102 mb).

### Data at $|t| \leq 0.5 \text{ GeV}^2$ .

In accordance with the strategy laid down in the introduction, all parameters of Table 1, are fixed when fitting the amplitude (with its new parameters) to the  $t \neq 0$  data. The three nonlinear parametrizations ((i,ii,iii)) have been tested in both models "A" and "B" and compared to the linear ones.

As an example of our results, we show in Fig.4 the comparison of the theoretical angular distributions to the data in the first cone, in the case of model "A" with non linear component either in trajectory and-or- in residue giving a high quality fit (with  $\chi^2_{d.o.f} \simeq 1.8$ ). A special attention is paid in Fig.5 where the very small  $t$  region of the nuclear-Coulomb interference is exhibited. The relevant parameters are listed in Table 2. First, we compare the models "A" and "B" of the Pomeron and comment the contributions of the Reggeons. Then, we discuss and interpret the results obtained in terms of the residue functions and the trajectories for the Pomeron.

#### 1. Quality of the fits.

In all variants (i - iii) considered, the quality of description of data is slightly better for the model "A" of the Pomeron than for the model "B" (we draw such a conclusion from the  $\chi^2_{d.o.f}$ , but the plots are sometimes undistinguishable by eye). We failed to improve the model "B" by choosing residue functions of the "QCD" form  $(1 - \frac{t}{t_i})^{-4}$ .

## 2. Cross-over.

Account of the cross-over factor in the  $\omega$ -part of the amplitude (13) allows to improve the description of data. In fact, setting 1 instead of the factor  $1 + t(z_1 + z_2 \ln \tilde{s})$  and refitting the remaining free parameters increases the value of  $\chi^2_{d.o.f}$  by 30% in all versions under consideration. In addition we find the value of  $b_\omega$  to be quite relevant and we fix it equal to zero.

## 3. Secondary reggeons.

Owing to the fact that the low-energy angular distributions (down to 9 GeV, where the  $f$ - and  $\omega$ -Reggeons are very important) enter in the fit, we were able to adjust the slopes of the secondary trajectories  $\alpha'_f$  and  $\alpha'_\omega$ . The values we obtained are always noticeably larger than those commonly reported. The same observation has been made when eikonalizing amplitudes to describe  $\frac{d\sigma}{dt}$  beyond the first cone [14]. Maybe it is suitable to use nonlinear parametrization for  $f$ - and  $\omega$  trajectories as we do for Pomeron.

## 4. Nonfactorizability of the Pomeron.

A good description of the data for  $\frac{d\sigma}{dt}$  is obtained only if  $F_0 \neq F_1$  (or  $\beta \neq 0$ ) in (15). As already noted in Sect.3, this is an evidence of nonfactorizability of the Pomeron at available energies. Since  $\beta < 0$ , the deviation from factorizability is decreasing with  $|t|$ . We emphasize that we are concerned with the low  $|t|$  region and simple Pomeron alone (without rescatterings or cuts). At truly asymptotic energies, the factorization is restored (the leading term is with  $F_1^2(t)$  in (14),(17)). The observed effect of nonfactorization we claim has nothing to do with subleading contributions ( $f, \omega, \dots$  etc.) which are explicitly taken into account. Notice only that, contrary to DIS analyses, our conclusion rests on the extremely good quality of both data and fits.

## 5. Non linear effects.

An explicit form of the function  $\phi(t)$  appears to be largely irrelevant (we believe this is due to the small  $t$ -domain investigated here; choosing  $\phi_i(t)$  will be more crucial for large  $|t|$ , as well as choosing an exponential form (16) to define the residue functions themselves). However, we found that the free parameter  $\tau_0$  always tends to its smallest possible value  $\tau_0 = 4m_\pi^2$  value and thus we can fix it at this limit.

We observed that in the versions with a non linear function  $\phi(t)$ , the  $\gamma_P$  parameter tends to zero, *i.e.* the Pomeron trajectory tends to be a linear one. However, for versions with linear  $\alpha_P(t)$  and non linear  $\phi(t)$  and versions with nonlinear  $\alpha_P(t)$  and linear  $\phi(t)$  the difference

in  $\chi^2$  is less than 2%. Only when  $\alpha_P(t)$  and  $\phi(t)$  are both linear, the  $\chi^2$  increases by 20%. Therefore, in accordance with the theoretical arguments discussed in Sect.2, in favor of a nonlinear trajectory, we prefer the versions with nonlinear  $\alpha_P(t)$  and linear  $\phi(t)$ .

## 6. Slope of the Pomeron trajectory.

Another important and interesting result of our investigation is that we find quite a larger slope of the Pomeron trajectory than the "world value"  $\alpha'_P = 0.25 \text{ GeV}^{-2}$ . In the linear cases the results of fits gives even larger values. For the non linear cases, as already said, the slope is  $t$ -dependent. The calculation of  $\frac{d\alpha_P(t)}{dt}$  at the frontiers (for  $t=0$  see (26)) and at the center of the first cone are shown in Table 3, for the same choices of parametrization as above in Table 2. The trend with  $t$  is in accord with earlier investigations [15].

The conclusions of the previous points are valid for both models "A" and "B".

## 7. Pomeron trajectory and $2^{++}$ glueball.

We now present the results concerning the prediction of a  $2^{++}$  glueball obtained in the dipole Pomeron model with unit intercept ("A"). We consider the non linear trajectories with a threshold at  $t_P = 4m_\pi^2$ . These trajectories have an imaginary part at  $t > t_P$ . The behavior of the real and imaginary parts are shown in Fig.6 in the particular case of the three Pomeron trajectories listed in Table 2. We found that in all variants giving a similar  $\chi^2$ , the Chew-Frautschi plots show also similar predictions for the mass and the width of the resonance

$$1.89 < M_g(\text{GeV}) < 1.92,$$

$$100 < \Gamma_g(\text{MeV}) < 400.$$

One notes quite a good agreement with the result of the WA91 experiment [8b]. The measured values for the X(1900) which could be a single state with  $I(J^{PC}) = 0(2^{++})$  are

$$M_g = 1.926 \text{ GeV} \pm 12 \text{ MeV},$$

$$\Gamma_g = (370 \pm 70) \text{ MeV}.$$

Note that, among the various case of nonlinear trajectories examined here, the (i) case (18) involving a square root leads to the width in best agreement with the experimental value.

A larger glueball mass is predicted in [16], where the nonlinear Pomeron trajectories with an intercept larger than 1 is used to describe the data on the whole  $t$ -domain for which data exist.

## 5. Conclusions

To summarize, we emphasize once more that the nonfactorizable form of the Pomeron amplitude as well as the nonlinearity of its trajectory  $\alpha_P(t)$  or/and of the function  $\phi(t)$  entering in its residue is strongly suggested by the data. To clarify the question whether the nonlinear behavior of the first diffraction cone is due to a nonlinear trajectory or to complicated residue functions, it would be necessary to have more precise data at high energies. The projected measurements at RHIC and LHC energies (where the  $f$ -Reggeon contribution is expected to be negligible) will certainly bring useful informations and, possibly, a reliable answer.

An important result of this work is a larger than usually used effective slope of the Pomeron trajectory we encounter in all successful parametrizations. It decreases with  $|t|$  but for the first cone it varies within the limits:

$$0.32 < \frac{d\alpha(t)}{dt} \text{ (GeV}^{-2}\text{)} < 0.46, \quad \text{for} \quad -0.5 \text{ GeV}^2 < t < 0.$$

Finally, we point out that the trajectory parameters are found very close to each others in the various parametrizations. They yield a mass and width of the first  $2^{++}$  glueball very close to the X(1900) observed by the WA91 collaboration. In our opinion, one may interpret this as a confirmation of a possible determination of the true Pomeron from the study of small  $|t|$  data on the elastic hadron scattering.

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	Model A	Model B
$\chi^2_{d.o.f}$	1.13	1.15
$gf$	-24.5	-15.8
$g_\omega$	9.16	9.29
$\alpha_f(0)$	0.808	0.714
$\alpha_\omega(0)$	0.451	0.445
$g_0$	7.98	6.68
$g_1$	-1.48	-8.59
$\Delta$	0	0.064

**Table 1**

Parameters of the scattering amplitude for the  $f$ -Reggeon (12),  $\omega$ -Reggeon (13) for versions "A" (double pole, (14)) and "B" (supercritical, (17)) of the Pomeron, fitted to the  $t = 0$ -data.

Parameters	(i)	(ii)	(iii)
$\alpha'_f$ ( GeV <sup>-2</sup> )	1.27	1.29	1.25
$b_f$ ( GeV <sup>-2</sup> )	0.443	0.387	0.499
$\alpha'_\omega$ ( GeV <sup>-2</sup> )	0.980	0.975	0.967
$b_\omega$ (fixed)	0.	0.	0.
$z_1$ ( GeV <sup>-2</sup> )	-6.01	-6.22	-5.77
$z_2$ ( GeV <sup>-2</sup> )	2.62	2.68	2.55
$\alpha'_P$ ( GeV <sup>-2</sup> )	0.256	0.288	0.294
$\gamma_P$	0.027	0.022	0.044
$\beta$ ( GeV <sup>-2</sup> )	-2.23	-2.33	-2.21
$b_0$ ( GeV <sup>-2</sup> )	5.32	5.47	5.32
$\gamma_0$ (fixed)	0.	0.	0.

**Table 2**

Parameters of the scattering amplitude for the  $f$ -Reggeon (12), the  $\omega$ -Reggeon (13) for version "A" of the Pomeron (double pole model "A", in the three cases of non linear trajectory (18)(20)(22) and linear residue (28)) fitted to the  $|t| \leq 0.5$  GeV<sup>2</sup>-data.

For each case ((i),(ii), (iii)), the two other possibilities involving a nonlinear component in the trajectory and/or in the residue ( $\gamma_P$  and  $\gamma_0 \neq 0, \gamma_P = 0$  and  $\gamma_0 \neq 0$ ) give a same  $\chi^2$  within a few percents and are undistinguishable on curves.

Eq.	$\left[\frac{d\alpha_P(t)}{dt}\right]$ $t = 0$	$\left[\frac{d\alpha_P(t)}{dt}\right]$ $t = -0.25\text{GeV}^2$	$\left[\frac{d\alpha_P(t)}{dt}\right]$ $t = -0.50\text{GeV}^2$
(18)	0.458	0.358	0.335
(20)	0.438	0.339	0.320
(22)	0.438	0.338	0.321

**Table 3**

Effective slope of the Pomeron trajectories (in  $\text{GeV}^{-2}$ ). Only model "A" parameters of Table 2 are used at the center and the limits of the first cone.

## Figures captions

**Fig.1** Cross-over of the experimental  $pp$  (triangles) and  $\bar{p}p$  (circles) angular distributions at 13 GeV (dashed and solid lines respectively are interpolations of the data). When the energy increases, the  $|t|$ -value of the crossing point goes smoothly below .15 GeV<sup>2</sup> (see the text).

**Fig.2** Fits of the total cross-sections up to the Tevatron energy as calculated with version "A" for the Pomeron (see the text and Table 1).

**Fig.3** Fits of the ratios of the real to the imaginary part of the forward elastic amplitude as calculated with version "A" for the Pomeron (see the text and Table 1).

**Fig.4** Angular distributions limited to the first cone (a factor of  $10^{-2}$  between each successive curve is used). The Tevatron data ( $\bar{p}p$  at 1800 GeV) are not included in the fit. The solid curves are calculated with model "A" and include a non linear component for the Pomeron either in trajectory and-or- in residue (see Table 2 for the parameters).

**Fig.5** Enlargement of fig.4 showing the (very small  $|t|$ ) nuclear-Coulomb interference region.

**Fig.6** Real and imaginary parts of the trajectories versus the four-momentum transfer for the various options of the Pomeron dipole with  $\delta = 0$  (model "A") discussed in the text. The solid lines correspond to (i); dashed lines to (ii); dotted lines to (iii). All these Chew-Frautschi plots predict a mass and a total decay width of the first candidate glueball with  $J^{PC} = 2^{++}$  in agreement with the measurements of the X(1900) by Wa91 Collab. [8b] (slight differences would occur in predictingg the next recurrence ( $J = 4$ ), but the predicted order of magnitude is the same).

The lower part of the figure exhibits the non linearity of the real part of the trajectory at low  $t$  (over a range symmetrical to the first cone).













